CHAOS AND THE COMPUTER

Farewell lecture, given at the Erasmus University of Rotterdam

on Friday, 9th of September 1988

by prof. Dr. J. Verhoeff

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FOREWORD

Confined to my home during the spring of 2020, I watched chaos unfold all around me. Koos Verhoeff had taught me about fractals back in the 80's. That revelation and its wisdom brought still brings me beauty and perspective in this challenging time.

In 1988 Koos gave his farewell lecture, titled "Chaos and the Computer," at the Erasmus University of Rotterdam. It proved not only insightful, but prescient. I'm pleased to share a translation of this lecture as well as some of Koos', and my own, fractal designs.

Thank you, Koos.

Anton Bakker

Chaos and the Computer

Ladies and gentlemen, I wanted to use this farewell lecture to leave a message, or, if you like, to ring the alarm. I don't mean the cutbacks that plague the university, they are just a ripple on the surface. That's why I changed the original title, "**Chaos and the University**", because I was afraid people would think I was going to talk about policy. No, it is all about a revolution in scientific thinking that has taken place over the past twenty-five years. Fractals play an essential role in these developments.

This will be discussed in the first part of this lecture, as the rest could not be understood without comprehending what is for many a new, important concept.

Before I start, I need to explain what my role in this is. I did not invent fractals and I did not participate in the revolution. Over the last ten years I followed the development of fractals with interest without initially being aware of the deep implications, implications that will be discussed in the second part of this lecture.

The treacherous thing about fractals is that they themselves are incredibly beautiful and offer surprising insights into the structure of often-chaotic nature. It is not uncommon for fractals to be associated with art.

Anyone who knows me even a little will not be surprised that I am mainly interested in the artistic aspects of fractals.

A great mathematician once said that beautiful mathematics was also good mathematics. Be guided by your sense of beauty. Another mathematician once said that good mathematics is also applicable in the long run. Ergo: beautiful mathematics can eventually be applied, a kind of depth investment in beauty.

I believe that the same applies to computer science, but it's a pity that people are so focused on profit nowadays, or at least think they should be.

That’s why I was pleased when I saw an article early this year in which fractals were used to compress data.

It should be noted that knowledge of fractals is due to the general availability of the computer.

I only gradually realized that fractals make it clear that certain systems show a fundamentally unpredictable behavior. This means, among other things, that more information about the current situation does not lead to better predictions.

My first thought was: my model-loving colleagues should know about this. Maybe they know the law, but I missed that. My second thought was: the politicians should know about this. They always use the results of the models of the CPB [Central Planning Bureau], the CBS [Statistics Netherlands] and the like so confidently as justification for their policy. My third thought was: everyone should know about this, because citizens are being misled by those political statements.

Dear audience, I know there will be many of you who will not like this message. After all, we grew up in a culture in which the pursuit of ever-increasing precision was seen as a great virtue. Some may be comforted by the thought that mathematicians have struggled to construct a few pathological systems with these unpleasant properties. I have to disappoint them: most systems are afflicted. And I have even more bad news.

The ancient belief that minor deviations have minor consequences is not correct!

Finally, the widespread confidence that systems tend towards a balance is also incorrect.

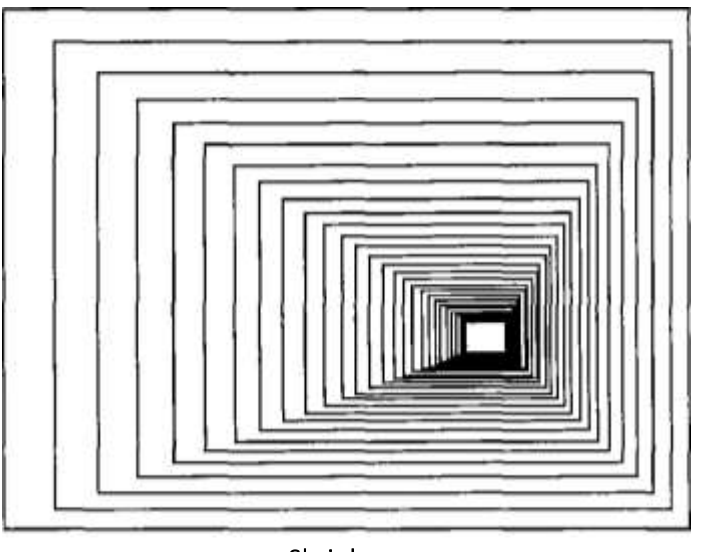
But let me first explain the why and how.

The Fractals!

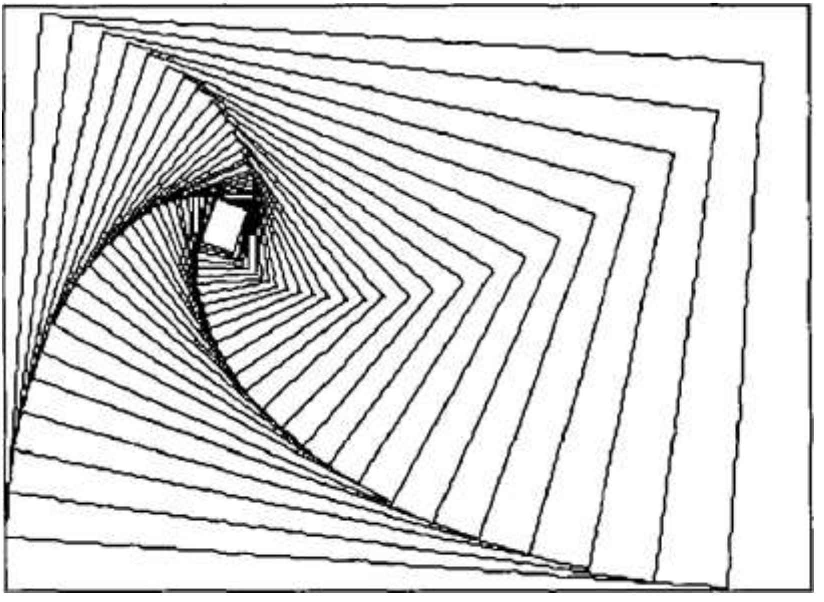
Fractals are mathematical structures with the property that certain parts, no matter how small, are a (reduced) copy of the whole. In stochastic fractals, the parts are not an exact copy, but resemble the whole, just as two snowflakes look alike without being the same.

The term "fractal" was only introduced by Benoit B. Mandelbrot in 1975, although the first fractals have been around for almost a century.

Fractals are all around us. A homely example of a fractal is the nurse on the Droste cocoa can [who holds a can with her image on it, which is holding a can with her image on it, etc.]. The infinitely repeating pattern is called a fractal. Each can on a can is the shifted and reduced image of that can.



Shrinkage

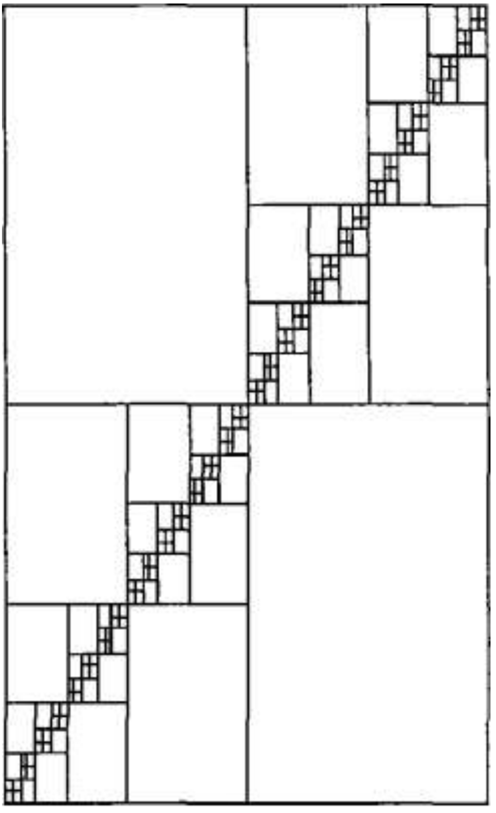


Shrinkage and turn

The cans on the cans disappear, as it were, into a single point. The nose tips of the nurses also do that.

A new world opens up when the nurse holds the can to one side. The reduced copy is also rotated, a 'shrink-turn shift'. The nose tips then spiral to a vanishing point.

It becomes even more interesting when each nurse holds a can in each hand. Each subsequent generation is then twice as numerous as the previous one.



The nose tips swarm out to a line segment when two cans are offered.

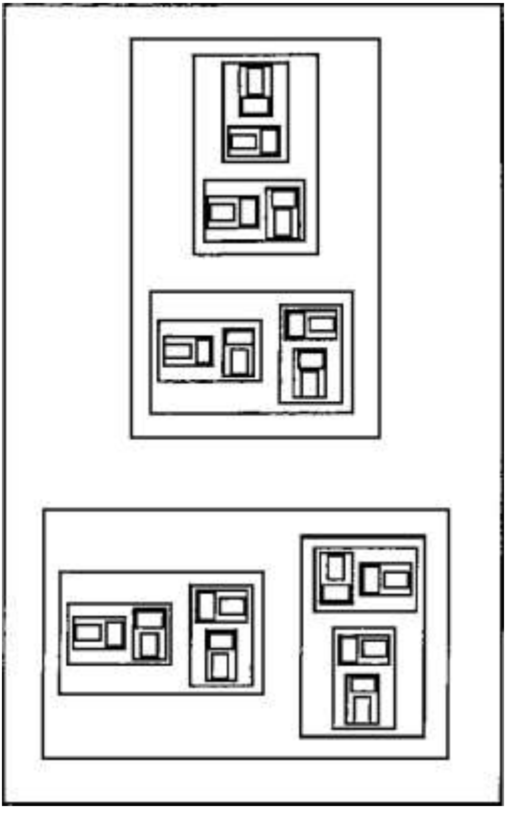
The collection of nose tips is also a fractal.

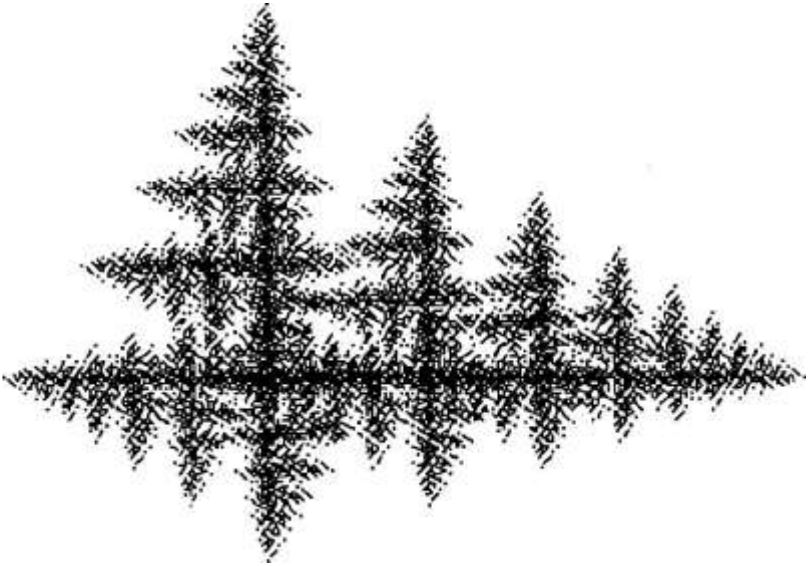
If you choose to reduce by a factor of three, one of the oldest fractals is created, namely the Cantor collection. This can be created by removing one third of a line segment from its middle and doing the same thing with the two remaining segments.

The Cantor collection is an example of a material fractal. A material fractal is so incoherent that every piece under the magnifying glass turns out to consist of different parts. If one reduces the Cantor collection from one of the endpoints by a factor of three, the result is equal to one third of the fractal. The collection is also created when the two reductions are repeatedly applied at random points, in any order possible.

One also gets a good picture if one tosses a coin to decide whether to apply the one or the other contraction.

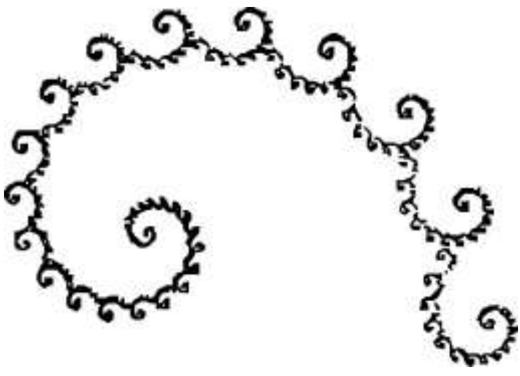
Certainly surprising is the nose picture of the nurses who have a can in each hand, one of which is on its side (see the next page.)





The nose picture, above, is a material fractal that has been created by using a shrink turn (0.7 and 90°) and a shrinkage (0.7). (Lauwerier, page 77)

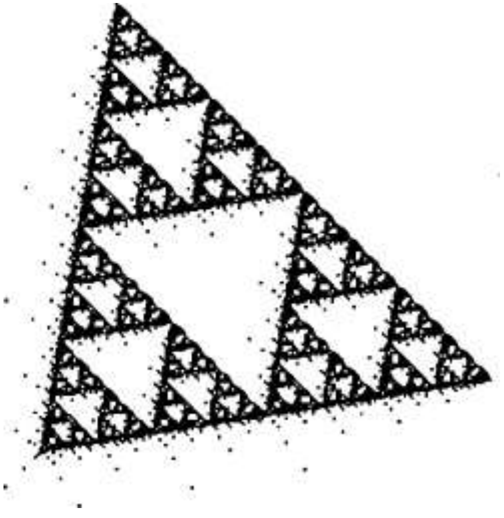
The nose-line picture below is created when both cans are tilted slightly. The spirals, made up of smaller spirals, seem very beautiful.



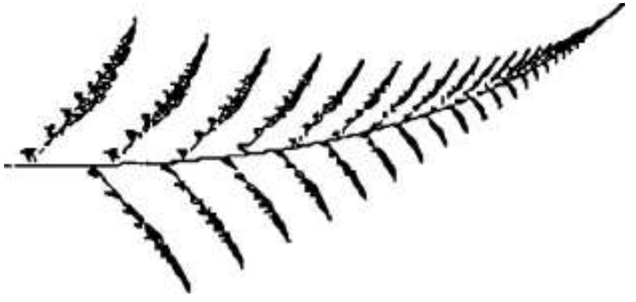
Here is another example from the book “Fractals”, by Hans Lauwerier, in which the various possible transformations are discussed in detail.

This is like the previous one, but now with a shrink turn of 0.85 over 45° and a shrinkage of 0.53. (Lauwerier, page 78)

Ladies and gentlemen, although nature has given nurses only two hands, we can just forget about this limitation and see what happens to creatures with three hands. Here you will see the material fractal that shows the noses when each nurse offers three cans (with three nurses).



It is some kind of two-dimensional Cantor collection, created by removing a triangle from the middle of the triangle and repeating it on a smaller scale with the three remaining triangles. This fractal can also be made by drawing lots to choose from three transformations, namely shrinkages, by a factor of two, relative to the vertices.

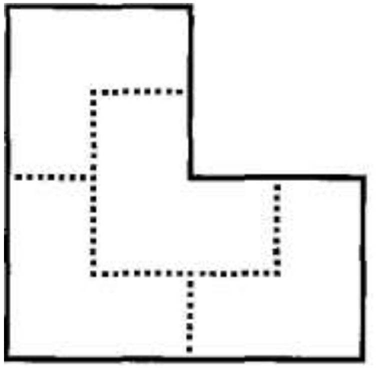


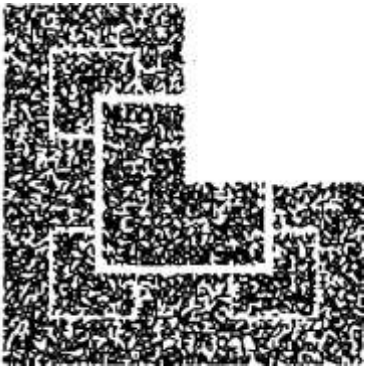
Now, two more examples of material fractals made by using four transformations.

The above picture, resembling a fern, is described in the article by Barnsley and Sloan [“A Better Way to Compress Images’].

This article mentions the so-called Collage Theorem. The theorem says that if one can cover any shape with reduced copies of this shape, there is a fractal with precisely that shape.

Examples:



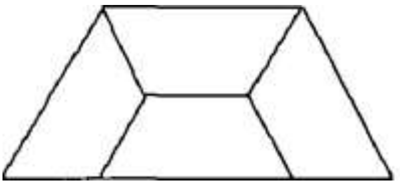


The puzzle with solution.   The corresponding fractal

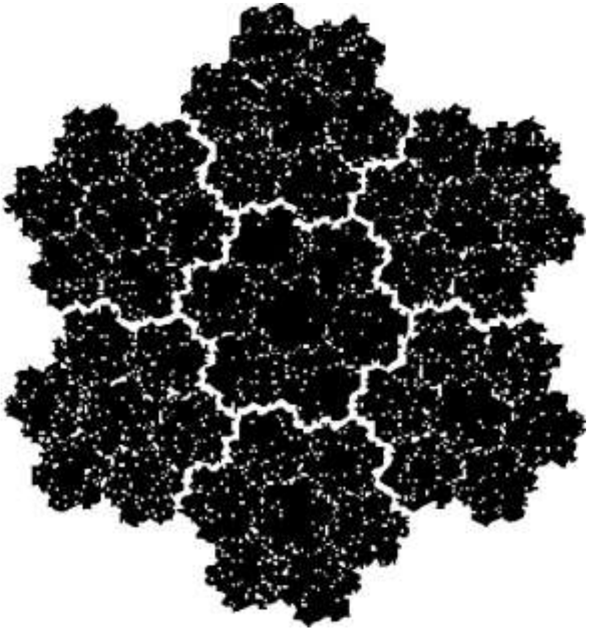
As a joke I smuggled in some cracks.

The first example is an application of the well-known puzzle about dividing three quarters of a square piece of land into four identical pieces, you know that legacy.

The second example is about the well-known trapezoidal conference table puzzle.

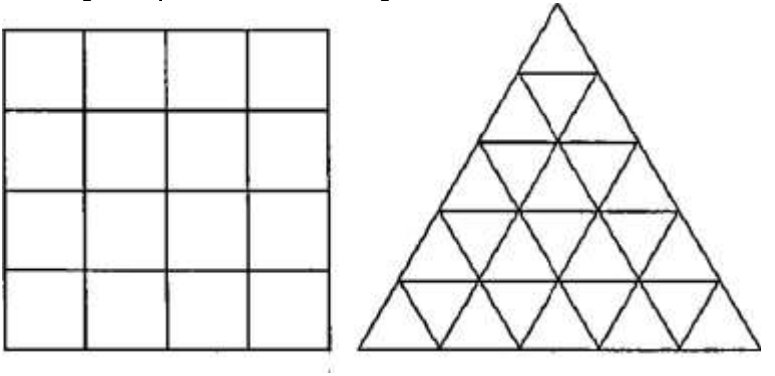


The third example concerns a shape that is covered by seven copies.

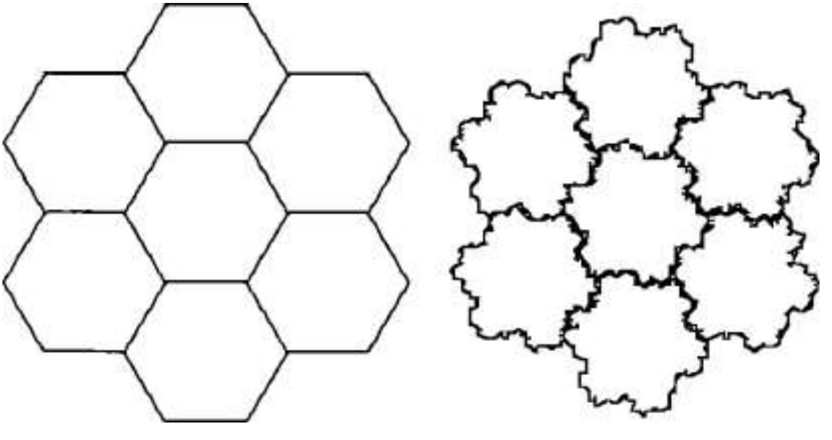


The cracks look like lightning flashes, which is not surprising, since they are also fractals.

As is known, squares and equilateral triangles can be used to build larger squares and triangles.



This is not possible with the third regular surface filler, the hexagon, but is with this squiggle slice!



It will be clear that some fractals have a “natural” appearance, but Barnsley states that a given natural form has a fractal with that shape. He makes a collage through a program with a given (natural) shape. He "fractalizes" such shapes.

The next leaf is completely determined by only twenty-four numbers, which is a lot more economical than indicating which of the sixty thousand dots are black. In the case of a picture of a circular disk, it would also suffice to specify the radius and the center.

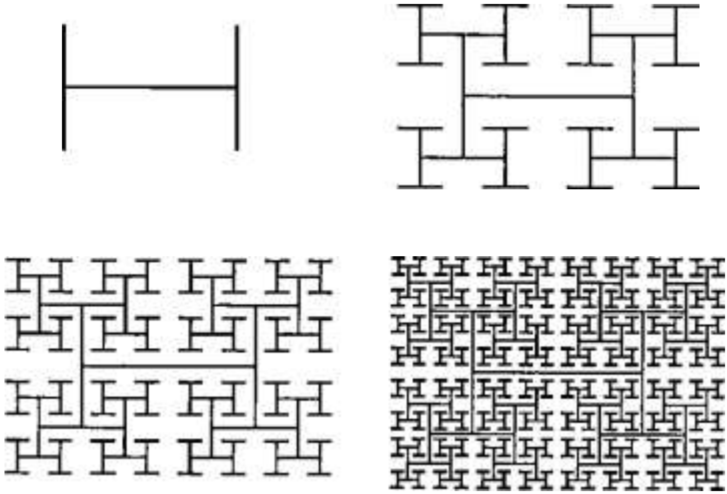
The exciting part is that the fractals apparently reveal a structure comprising otherwise quite chaotic manifestations in nature. After all, no two beech leaves are alike and yet we humans recognize them. We recognize a Chinese person as such even though we see him for the first time. All in all, exciting new possibilities for pattern recognition.

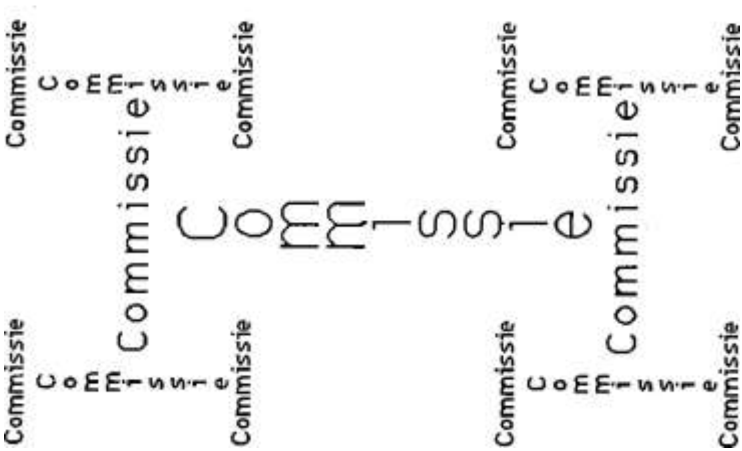
I'm eagerly looking forward to Barnsley's forthcoming book, "Fractals Everywhere."

Nobody who has seen such images will be surprised that shapes, created by a growth process, can be well described with fractals.

Now some examples of related fractals. The figure below shows the various stages of the so-called H-fractal. Each H has a (small) H on the four ends, half the size. The actual fractal is created when one continues this process "infinitely." Yet the approximations are also called fractals.

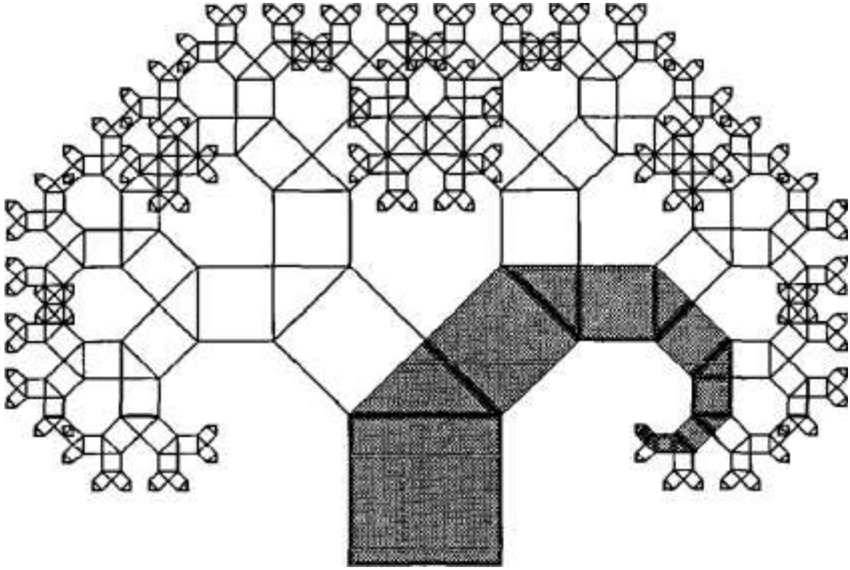
With a little imagination, one can see a Droste's nurse handing out copies with four arms outstretched. Each subsequent generation is three times longer.





Above you will see something similar to the organizational structure of the university.

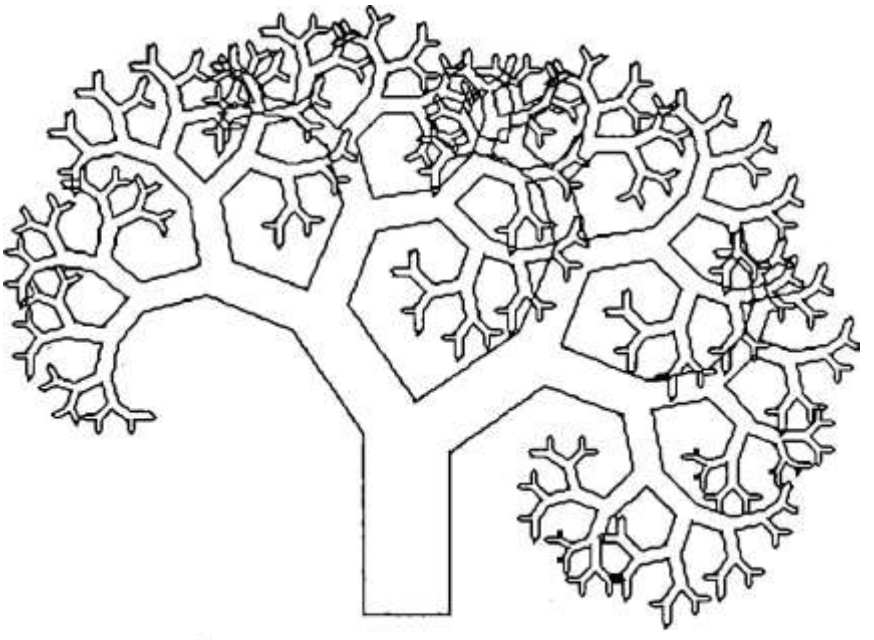
The shrink turn is used for the 'well known' Pythagoras trees, which creates spirals.



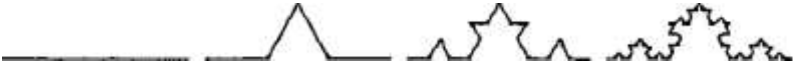
The continuous turning is clearly visible in the grey branch.

For all kinds of variations on the theme, I refer to the booklet "Bomen van Pythagoras" ("Trees of Pythagoras") by Bruno Ernst and to "Fractals" by Lauwerier.

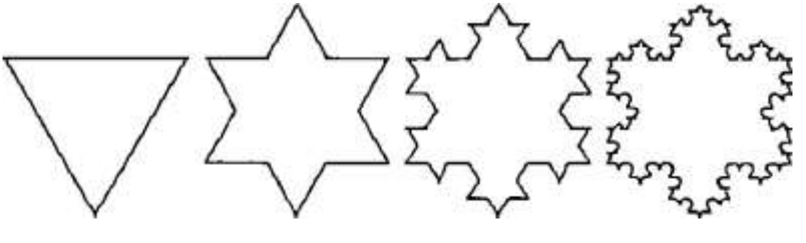
Below is another crooked tree.



Another example of an old fractal is the so-called Koch curve. This is created by replacing a line segment with four-line segments, as in the drawing below. Subsequently each of these four-line segments is replaced again by four (smaller) line segments, and so on.



By treating the sides of a triangle in this way, the so-called Koch island is created.



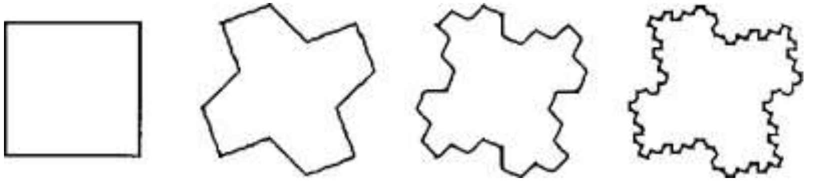
The remarkable thing is that the 'length' of such curves cannot be determined: they are, as it were, infinitely long. After all, every time you ‘walk around’, the curve becomes 4/3 times longer. The more accurately one measures, the longer the 'length' of the curve becomes. The area of the Koch Island is 60% larger than the original triangle.

Such infinite length goes against our intuition. After all, an estimate of the length of a circle improves if we measure it in smaller steps. In practice, however, surveyors who wanted to determine the length of a shoreline or river noticed that the measured length increased when smaller steps were taken. After all, if one does that, one has to walk around inlets and peninsulas, which one initially skipped. Viewed more closely, the inlets themselves appear to have inlets again. The details resemble the whole, typical of a fractal. It is amusing to know that Portugal sets the full length of its border with Spain at 1214 km, while Spain states 987 km for the same border. According to the Netherlands, the border with Belgium is 380 km, but according to our southern neighbors, the same border is 449 km. The moral is: "The smaller the country, the longer the border."

Around the turn of the century these types of curves were constructed as examples of continuous curves that did not have a tangent line anywhere. They were considered as products of a sick mind, not occurring in nature. They were also called the Monsters of Osgood. Only now is it recognized that in nature they are the rule rather than the exception.

There are endless variations in the formation of fractals à la Koch. One of these is the way Minkovski replaces a line segment with three line segments, as is illustrated in the figures below.

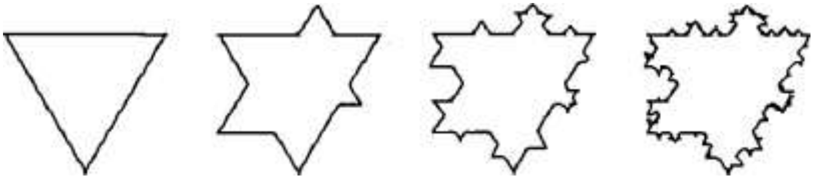
The sides of a square have been treated according to the above prescription. The Minkovski island formed that way has the property that every generation has the same surface area. The length of the outline is of course infinite again.



A surprising image is obtained when one walks around via one point centrally above the connecting line, provided that the distance to the center has been adjusted appropriately.

Now an example of a stochastic fractal.

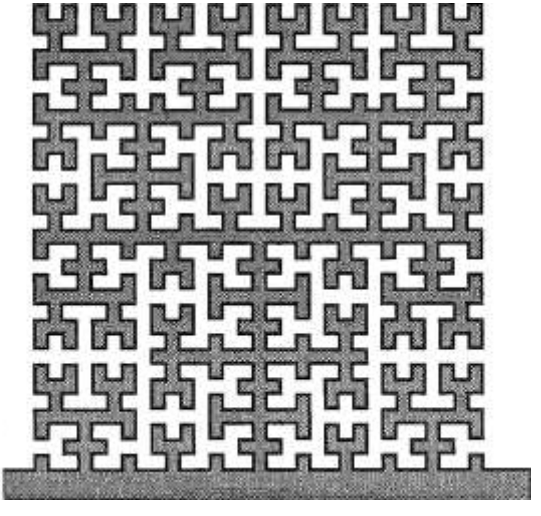
With a stochastic fractal the 'walking around' is estimated not exactly, as with Koch, but about at one third. The first estimate determines the overall shape, the subsequent estimates have less influence, and the rest is messing around in the margin. The pictures below give an example.



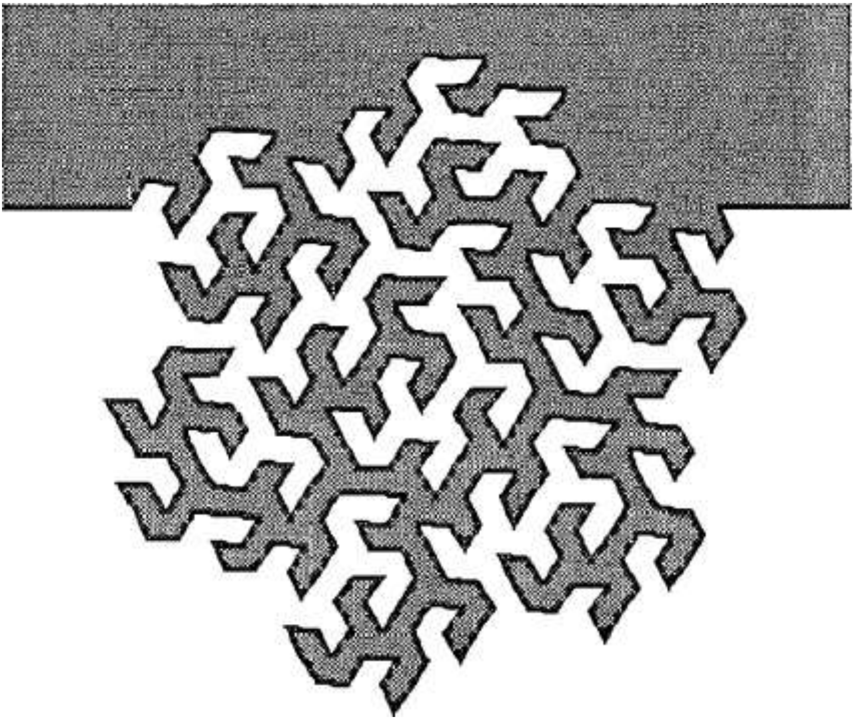
The final figure looks much more like a natural island. A stochastic fractal is therefore a better model for a coastline than a 'smooth' curve.

However, it can be even crazier.

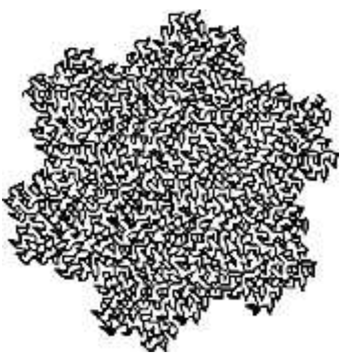
The Hilbert Curve can be seen as the coastline of Grey Land with a White Sea. The whole square is, as it were, coast (a kind of swamp?). A nice design for a marina?



Below is a picture of the so-called Gosper flowsnake. This is also surface-filling.



You can clearly see the shape of the ‘swamp’ in the following picture that shows the fourth generation [of fractal reduction].



It’s the same shape as the squiggle slice on page 12.

There is a measure for the 'tortuosity' of a curve: the 'smooth' ones have size 1, while the surface-filling curves (see Hilbert’s harbor) have size 2. The Koch curve is in between and measures 1.261.8 ... as a size. The coast of England has a 'tortuosity' of about 1.22 ....

Esteemed members of the audience, the fact that fractals model nature well is evidenced by the employment of the computer in drawing natural-looking scenes using stochastic fractals.

These scenes include landscapes, planets, islands, clouds, all realized with computer-controlled simulators.

Products may also be created that are the result of a growth process.

It would be possible to give all kinds of technical products a more natural appearance with the fractals. One often does not realize that modern optimization efforts, a typical computer product, can lead to uniformity. This is reinforced by the increasing use of plastics. The technology creates a ‘sleek’ environment with a lot of predictable regularity (for example terraced houses and flats), while nature (and the coast) treats our senses to variations on a specific theme, which is recognized by us people, pattern recognizers par excellence.

Ladies and gentlemen, after this long introduction we have come to the main part, namely the models and the fractals themselves.

In 1845 Verhulst drew up a model for a population with limited living space. Say x is the part of this space that is populated, then 1-x is the unoccupied part.

Verhulst assumes that growth will be proportional to the size of x, with a fertility factor a and with the available unoccupied living space. So if X_{n+1} is the next generation and X_n the current one, the following applies: $X_{n+1} = a \cdot X_n \cdot (1-X_n)$ (with a less than 4, otherwise $X>1$).

By repeatedly calculating the next value from the previous one, a series is created. The idea behind this model was that this series should aim for a final value.

For an unlimited growth $X_{n+1} = a \cdot X_n$ would apply, this is an uninteresting model as the population would grow unrestrained (interest on interest).

At first glance, Verhulst's model appears to be simple, as a stable condition is obtained for $X = 1-1/a$.
After all: $(1-1/a) = a.(1-1/a).(1- (1-1/a)) = 1-1/a$.

Take for example a=2 and you can see that there is nothing to do for X=1/2. After all, if $X_n=1/2$, then $X_{n+1}=1/2$ also applies, and stability occurs. Regardless of the initial value, if greater than zero, balance is reached at 1/2. This is exactly what one should expect of a decent model.

However, the curious thing is that the model becomes indecent if the values for a are greater than 3. Admittedly $X=1-1/a$ provides balance, but it is not stable yet. With another initial value, no matter how close to 1-1/a, the X moves away from 1-1/a and will eventually jump between two values, z0 and Z1; with z0 = a.z1.(1-z1) and z1 = a.z0.(1-z0). This is called a two-cycle.

If the value for a is allowed to grow even further in small steps, a very surprising effect occurs. The two-cycle becomes unstable and is replaced by a four-cycle. Then this one becomes unstable and is replaced by an eight-cycle. This is called a period doubling or bifurcation. As long as a is less than 3.569946... this doubling will continue to occur. Typically the behavior of a fractal. However, if one goes past 3.569946 ... there is no trace of periodic behavior, but there is complete chaos. The x will jump all over the place.

It will be clear that the gigantic amounts of calculations could only be performed with a computer or a programmable pocket calculator. The latter is exactly what Mitchel Feigenbaum did in the early 1970s.

The chaotic behavior of the population has the character of a fractal. This means that if you look at the value of a in more detail, you will encounter the same kind of chaos on a smaller scale. So, with more information about the value of a, one cannot get better predictions about the future behavior of the system.

Something similar applies to other, less simple dynamic models. A well-known example is the weather. The dream of meteorologists, to be able to predict the weather more accurately with more information and larger computers, turns out to be a dream.

As early as 1962 this was discovered, accidentally, by the meteorologist Lorentz. A computer simulation of a simple meteorological model showed that the result changed significantly due to a minor error when entering the initial state. His publication in a meteorological journal went unnoticed for more than ten years by mathematicians, physicists and certainly by economists.

Ladies and gentlemen, I believe in visual education and would therefore like to explain this with a mechanical model. Here you see a spatial pendulum with a bicycle bearing. In addition to gravity, there are three magnets exerting their influence. As you can see, the pendulum has three stable rest positions for the magnets.

There is also an unstable rest position exactly in the middle of the magnets. Of course there is friction and air resistance, otherwise the pendulum would go on forever, something that is hardly suitable for a demonstration at a farewell. If we ignore the turning of the earth, the position of the moon and the tram on the Oudlaan and hundreds of other relevant matters, such as your breathing and that annoying fly, it is a simple model. The pendulum's behavior is a bit strange, as you can see when I give it a nudge.

You can see it hesitating between red and blue and then suddenly, left from the flank, it turns to green. Kind of like two dogs fighting over a bone. In theory it is clear that it is computable where the pendulum will come to rest if it is released from a certain point, at zero speed. Suppose this is done for every square millimeter of this surface. This is approximately 16 by 16 cm, or 25600 mm2, and therefore a lot of calculations. Nowadays that is not a problem.

Suppose we color the boxes red, green or blue depending on the color of the winning magnet. If the pendulum ends in the middle position, we will color the box black. This creates a picture in which the area around the red magnet is of course red and we also have green and blue zones around the other magnets and possibly a black box here and there.

If we then try to make predictions with this map, we have a problem. If I let the pendulum start on a red box, it turns out that it is not very reproducible. What is going on? It turns out that it is important where exactly I start in the box. If I take a closer look at the red box and, for example, divide it into one hundred boxes of a tenth of a millimeter and calculate for each of these where the pendulum will end, then it appears that all colors can occur again. This of course depends on the location of the box, because with a typical red box near the red magnet everything remains red. In most places, however, all colors reappear. You guessed it, we have another fractal.

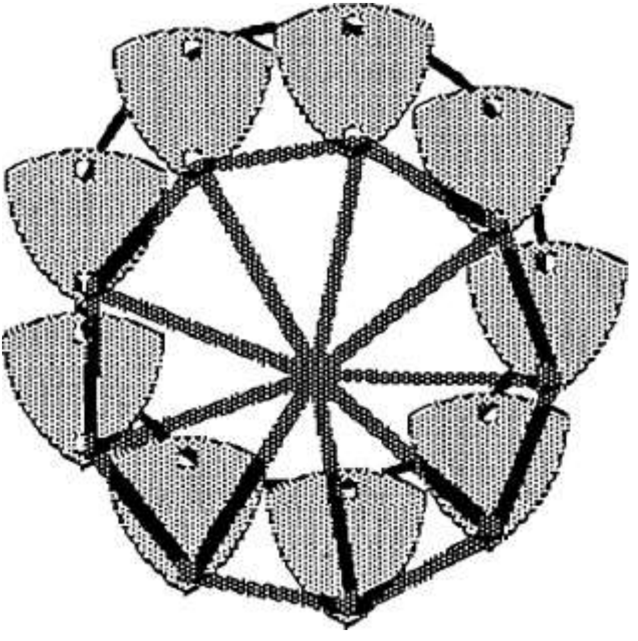
So no matter how much we increase the accuracy, we always get a mixture of all colors. Therefore, it is basically unpredictable where the pendulum will end. Whenever the starting position is given more accurately, one may get a different final position. This applies to the neat, idealized model.

If we make the model more complicated, for example by taking the moon's gravitational pull into account, the model becomes more complicated, but the unpredictability remains.

Dear audience, you have no doubt wondered what is under the sheet to my left. I expected that and that's why it is there. I believe that one should stimulate the audience to ask questions, one should make them curious, every lecture should be an adventure and a farewell lecture should be no exception.

Underneath is another mechanical model that I would like to demonstrate.

Look, this is a wheel with containers that can turn (give it a nudge and it will turn). Here is a faucet that can fill water into the top container. This makes the container heavier and it turns away, so that the next container is filled with water. As you can see, the wheel will start to turn. However, the trick is that the containers are leaking and therefore lose water, so that the first container is empty by the time it is back up. It is filled again and so we continue. This is of course not a perpetual motion machine and it is also not an invention for running cars on water. However, if I now accelerate, by opening the tap further, the wheel, as you can see, will indeed turn faster.



But now the cat's out of the bag. The top container does not have enough time to drain all the water, which slows down the movement. So, a little more water and the behavior of the wheel can safely be called chaotic and unpredictable.

The point is that almost all nonlinear dynamical systems show such chaotic behavior. It is now known that the planetary movements, the cosmological clockwork par excellence, will show a chaotic character in the long run. In the long run means in this case maybe millions of years. However, in case of weather forecasts it is about 24 hours.

One of the challenges for the Faculty of Economics is to investigate the period in which the economic models are predictable. It seems absurd to me to assume that such models can make some useful statements about, for example, the number of unemployed in 1992.

As I said before, however, the most annoying part is that politicians hit us every day with such forecasts.

I dare say that it is the university’s duty to let society know the possibilities and certainly also the impossibilities of science. In my opinion, the argument that the latter is not possible because 'people' will not understand it does not hold, since the university has not delivered the message correctly.

Yet there are even publications in which scientists suggest that with knowledge of fractals better predictions can be made about the weather or the stock market. It is similar to deliberate deception, even if its purpose, to stimulate research into fractals, is a good one.

In his book "The Structure of Scientific Revolution" Thomas Kuhn notes that the scientific establishment often tries to minimize or even ignore revolutionary new insights.

The young researchers passionately throw themselves at the new insights, but the old guard, who became famous in the maligned paradigm, is unwilling to applaud them.

In any case, this applies to the fractals and the consequences I have outlined. It's perfectly fine to talk about the beauty of the fractals or about the possibilities of uncovering their secrets for nature, but nothing is said about the impotence of science when it comes to predicting the future.

It is true, scientists are not entirely at fault. After all, it is society itself that increasingly expects that the scientific pioneers can justify their research with social relevance.

In summary, I mention three things that we will have to get used to.

1) More information about the current state of a system will often not lead to better predictions.

2) Minor changes in the model may have major consequences for the results. It is said that a butterfly in Beijing can cause a hurricane in Texas.

3) Many systems do not tend to be in a state of equilibrium, but often display chaotic behavior.

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This may seem like a bad message to those striving to control the future, but I think it is a good one in the end. After all, it holds the promise that life will remain full of surprises and therefore interesting.

I finish with a quote from Professor Tromp that I heard over the radio while editing this text: "Most important things happen completely unexpectedly."

Ladies and gentlemen, thank you for your attention.

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